Sizing control valves for non-turbulent flow by means of the loss coefficients $K$ and $K_L$

There are a few methods available for calculating the flow through a control valve under non-turbulent conditions. The two receiving the stamp of official international recognition are those described in the IEC standard 60534-1-2 and the ISA/ANSI standard 75.01 (refs 1 & 2). Why then is it important to introduce another method to add to the process control engineers’ libraries? The answer is that there is no call for another method but over many years the author has toyed with the idea that the valve’s Reynolds Number Factor ($F_R$) can be calculated quite simply through the ratio of the turbulent and laminar loss coefficients $\sqrt{K/K_L}$. The Reynolds Number must be known but no other information is required. The method does not require any references to graphs. Unfortunately the method proved not to be quite so simple. Changes in flow patterns at low Reynolds Numbers and some uncertainties about the values of the loss coefficients ($K_L$), also at low Reynolds Numbers, brought the work to a standstill until, with some reluctance, the problem was resolved by the introduction of that panacea for all ills, a modifying factor $f$. It is hoped that as flow technology develops, particularly in the role of loss coefficients for non-turbulent flow, these problems can be analysed and resolved technically, thus eliminating the empirical correction factor.

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Introduction

The problem of sizing control valves for non-turbulent flows has received considerable attention since Gordon Stiles published in 1967 what was possibly the first paper addressing this subject. Since then there have been a number of contributions from leading engineers including Dr. Jorg Kiesbauer (Ref 3) and Dr. Hans Baumann (refs 4, 5 and 6). The latter has without doubt introduced the most significant contributions to this subject and the IEC and ISA/ANSI standards owe much to his work. These developments in non-turbulent sizing methods are clearly reviewed in a paper by J.A. George: “The evolution and status of non-turbulent flow sizing for control valves.” (ref 7)

The Reynolds Number Factor $F_R$

In fully developed laminar flow the propulsive force at the controlling orifice is a function of the pressure drop ($\Delta p$), unlike turbulent flow where the propulsive force is related to ($\sqrt{\Delta p}$). If, considering all flow regimes the propulsive pressure drop is represented by ($\Delta p)x$, then from fully developed laminar flow (very low Reynolds Numbers) $x$ varies from 1 to 0.5 at turbulent flows (high Reynolds Numbers). The precise values of $x$ for the intermediate flow regimes are very difficult to determine, which is possibly one of the reasons which prompted Gordon Stiles to introduce the Reynolds Number Factor ($F_R$). This enabled a valve to be sized for non-turbulent flow assuming a pressure drop of ($\sqrt{\Delta p}$) as for turbulent flow. $F_R$ is defined as the ratio of the flow under non-turbulent conditions for a valve at maximum capacity with a pressure drop of ($\sqrt{\Delta p}$), to the flow with the same valve at maximum capacity under turbulent conditions with the same pressure drop ($\Delta p$).

$$F_R = \frac{Q_{\text{non-turbulent flow}}}{Q_{\text{turbulent flow}}}$$

* with the same valve positioned at the same $C_v$, and with the same $\sqrt{\Delta p}$. $F_R$ can be expressed differently in terms of the valve's rated $C_v$:

$$C_{\text{vl}} = F_R \cdot C_{\text{vt}}$$

where $C_{\text{vt}}$ is the normal rated $C_v$ of the valve with turbulent flow, and $C_{\text{vl}}$ is the rated $C_v$ of the valve with non-turbulent flow. In both cases the working pressure drop for sizing calculations is ($\sqrt{\Delta p}$).

In terms of the required $C_v$ for specific flow conditions $C_{\text{vl}} = \frac{C_{\text{vt}}}{F_R}$

For a given set of non-turbulent flow conditions, $C_{\text{vl}}$ is the calculated $C_v$ assuming turbulent flow. $C_{\text{vl}}$ is the required $C_v$ to pass the same flow but under non-turbulent conditions. The pressure drop associated with $C_{\text{vt}}$ and $C_{\text{vl}}$ is ($\sqrt{\Delta p}$).

Nomenclature

- $C_v$ = valve flow coefficient – USGPM
- $d_1$ = valve inlet diameter – mm
- $d_2$ = hydraulic mean diameter – m
- $d_e$ = valve trim effective diameter - mm
- $f$ = KL correction factor
- $F_P$ = piping geometry factor.
- $F_R$ = Reynolds Number factor
- $G$ = specific gravity
- $K$ = valve turbulent loss coefficient
- $K_L$ = valve non-turbulent loss coefficient
- $l$ = characteristic dimension in Rev - m
- $N$ = numerical constant
- $p$ = pressure - kPa abs.
- $\Delta p$ = pressure drop across valve - kPa
- $Q$ = volumetric flow rate – m$^3$/hr
- $v$ = velocity – m/sec = kinematic viscosity – m$^2$/sec or centistokes
- $\rho$ = density

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Using $F_n = \frac{C_{VLT}}{C_{VRT}}$ the numerator and denominator can be expanded:

$$C_{VRT} = (4.625 \times 10^4) \frac{d^2}{d}$$

$$(K_v, K_c, \frac{K_{NL}}{F_r}) = \frac{Nd^2}{\sqrt{K_v}} \frac{1}{\sqrt{K_c}}$$

divided by:

$$C_{VLT} = (4.625 \times 10^4) \frac{d^2}{d}$$

$$(K_v, K_c, \frac{K_{NL}}{F_r}) = \frac{Nd^2}{\sqrt{K_v}} \frac{1}{\sqrt{K_c}}$$

giving:

$$F_n = \frac{C_{VLT}}{C_{VRT}} = \frac{\sqrt{K_c}}{\sqrt{K_v}}$$

It should be noted that with fully developed laminar flow the pressure recovery factor $F_{LL} = 1$ but its value reduces to the turbulent values as the flow regime approaches the transitional and turbulent.

The modifying factor $f$

Compared with some actual results and with $F_n$ values calculated using the method recommended in IEC std. 60534-2-1 the $F_n$ values given by the simple term $\sqrt{\frac{K_c}{K_v}}$ are overstated, particularly at low Reynolds Numbers. This could be attributed to a number of reasons, all difficult to analyse and evaluate.

The three most obvious reasons are:

a) Reliable values for $K_r$ are difficult to find. Most of those used in this paper are as quoted in Miller’s book (ref 9). These are believed to include all the loss coefficients that affect the flow through the valve $\left( K_v, K_c, \frac{K_{NL}}{F_r} \right)$, but their compatibility with this system of working with loss coefficients is difficult to substantiate due to the lack of data from other sources. From published articles investigating laminar flow there is good reason to accept that $F_n=1$ at fully developed laminar flow. This will gradually change to turbulent values (1 → approximately 0.65) as the Reynolds Number increases (depending on valve style).

b) At very low Reynolds Numbers there is a change in the relationship between the resistance to flow at the valve trim and the resistance in the valve inlet section. The latter increases causing $K_v$ which relates to the pressure drop at the valve trim, to decrease. Also as the flow changes from turbulent to laminar the propulsive pressure drop at the trim changes from $\sqrt{\Delta p}$ towards $\Delta p$ accompanied by an increase in $K_v$.

c) It is possible for flow in one section of the valve to be turbulent whilst laminar flow may be sustained in another section.

All these and possibly other factors may account for $\sqrt{\frac{K_c}{K_v}}$ overstating the values of $F_n$. The values used for $K_r$ obtained from various sources, do not include any allowances for these variable changes in the relationship of the pressure drop at valve trim to that in the valve inlet section, as the flow changes from turbulent to laminar. Since quantitative information on the effects on $K_r$ of these changes is scarce, the introduction of a correction factor “$f$” must be considered.

The corrected value of $1/\sqrt{K_v}$ becomes:

$$\frac{1}{\sqrt{K_v}} \times f \quad \text{then} \quad F_n = \frac{\sqrt{K_v}}{\sqrt{K_c}} \times f$$

Using the few practical results available, accompanied by a generous amount of extrapolation along with comparisons with the values of $F_n$ as calculated from IEC 60534-2-1, reliable values have been apportioned to “$f$” enabling values of “$f$” to be calculated for a wide range of Rev values. For reference purposes the graph of $F_n$ against Rev drawn from IEC 60534-2-1 (ref 1) is shown in Fig 1. A similar graph but for valves with small trims is shown in Fig 2. It was found that values of “$f$” varied from around 0.5 at low Reynolds Numbers to 1.0 as the turbulent regime is approached.

Tabulated values of $K_r$, $K_v$ and “$f$” will be found in Figs.3, 4 and 5. Calculating $F_n$ using the Loss Coefficients

1) Calculate $C_{VRT}$ for the flow conditions being considered using the IEC sizing equation for liquid turbulent flow:

$$Q = 8.65 \times 10^{-2} C_{VRT} \sqrt{\frac{\Delta p}{A_1 / A_2}} \quad \text{(1)}$$

where $Q=$m$^3$/hr, $\Delta p=$kPa, $p=$kg/m$^3$

$F_p =$ is the piping geometry factor (ref to IEC std.60534-2-1). $F_p =1$ when the valve inlet and outlet connections are the same size as pipe connections.

Increase $C_{VR}$ by a value between 35% and 90% (150% for highly viscous fluids) to give an approximation to the larger valve rated $C_{VRT}$ required for the valve to pass the same flow but in the non-turbulent regime. Knowing the valve style required, select a valve and trim size with the nearest rated $C_v$ to this inflated $C_{VR}$. This actual valve rating for non-turbulent flow will be denoted by $C_{VR}$ in the following calculations.
Calculate $\frac{C_{VT}}{d^2}$ where $d^2$ is the valve inlet diameter (mm) and $C_{VT}$ is the nearest rated Cv (turbulent flow) to the $C_{VT}$ calculated from equation (1) but increased by 10% to 20%. The ratio is $\frac{C_{VT}}{d^2}$ required along with Reynolds Number for reading the tabulated values of $K$, $K_L$, and $f$.

For clarification:

- $C_{VT}$ is the calculated Cv for specific flow (turbulent).
- $C_{VL}$ is the calculated Cv for the same flow but non-turbulent. $C_{VL}$ is the valve rated Cv for turbulent flow.*
- $C_{VRL}$ is the valve reduced rated Cv for non-turbulent flow.

$C_{VL}$ is the rated Cv for a valve to be capable of controlling a specific flow in the non-turbulent regime.

* $C_{VT}$ is the valve Cv rating as published in the manufacturer’s literature.

$C_{VL} > C_{VT} > C_{VRL} > C_{VRL}$ is approximately $C_{VRL} \times (1.35 \rightarrow 2.50)$

1) For a control valve the Reynolds Number can be calculated using:

$$R_{ev} = \frac{273.76Q_{f_d}}{\nu \sqrt{C_{VRL} F_L}}$$

where $Q = m^3/\text{sec}$, $C_{VRL} = \text{USGPM}$, $\nu = m^2/\text{sec}$

or

$$R_{ev} = \frac{0.076Q_{f_d}}{\nu \sqrt{C_{VRL} F_L}}$$

where $Q = m^3/\text{hr}$, $C_{VRL} = \text{USGPM}$, $\nu = m^2/\text{sec}$

or

$$R_{ev} = \frac{76,000Q_{f_d}}{\nu \sqrt{C_{VRL} F_L}}$$

where $Q = m^3/\text{hr}$, $C_{VRL} = \text{USGPM}$, $\nu = \text{centistokes}$

Values for $K$ (turbulent flow) can be read from the tables but may also be calculated from:

when $\frac{C_{VT}}{d^2} \geq 0.016$  \hspace{1cm} $K = \frac{2.14 \times 10^3}{\left(\frac{C_{VT}}{d^2}\right)^{1.36}}$

when $\frac{C_{VT}}{d^2} > 0.016$ but $\geq 0.001$  \hspace{1cm} $K = 1 + \left[1.384 \times 10^3 \left(\frac{C_{VT}}{d^2}\right)^{1.36}\right]$ or $K = 1$ constant

The variation of $K$ with $\frac{C_{VT}}{d^2}$ is shown. The variation of the unmodified $K'$ with $Re$ is illustrated in Fig 7 (ref 9) and in Fig 8. Knowing the required values of $Re$, $K'$, $f$, and $\frac{C_{VT}}{d^2}$, $F_L$ may be calculated from:

$$F_L = \frac{K}{\sqrt{K_c} \times f \times \left(\frac{1}{F_L}\right)} \quad \cdots (2)$$

The $C_v$ for non-turbulent flow = $C_v$ for turbulent flow / $C_v$ for turbulent flow

In Fig 1 $F_i$ has been allocated the value of 1, but in Fig 1A (refs 7 and 10) the turbulent $F_i$ values have been used. Recent research has shown that this gives more realistic values for $F_i$. However it must be noted that at fully developed laminar flow, the actual flow is influenced by $F_L = 1$. As the flow regime changes from laminar through transient to turbulent $F_i$ approaches the turbulent value.

Conclusions

This method for sizing control valves for non-turbulent flow can be commended on two counts:

1) It is uncomplicated.
2) The results compare favourably with those calculated by the current methods and with current test data.

Unfortunately the technical argument cannot be resolved analytically because of the lack of precise data concerning loss coefficients and control valve flow patterns at low Reynolds Numbers. The solution at
the present stage must be the introduction of the correction factor “f” which in itself is difficult to evaluate using precise analytical methods, but the values tabulated in Figs 4 & 5, produced by pragmatic manipulation of a small amount of reliable data, yield calculated values of $F_R$ that compare favourably with the values given by IEC 60534-2-1. This loss coefficient method of sizing control valves for non-turbulent flow is presented as a simplified, but effective alternative to the current well established methods.